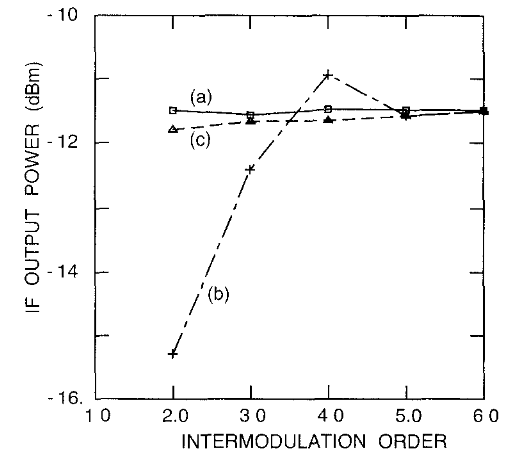

Fig. 4. Comparison of the simulated IF (50 MHz) output power versus the intermodulation order for the MESFET amplifier with two-tone input excitation. *a*: GPSA–AOM; *b*: APDFT HB method; *c*: dual-frequency-set APDFT HB method

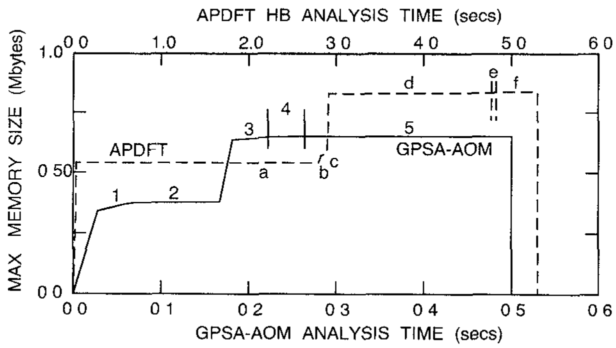

Fig. 5. Comparison of simulation run time and the maximum memory requirement for the MESFET amplifier with two-tone input excitation. The solid line is for the GPSA–AOM and the dashed line is for the dual-frequency-set APDFT HB method. The regions designated are listed in Table I. Computer run time is for a DEC DS3100 workstation.

IV. Conclusion

This paper presented a common error minimization algorithm for performing both the harmonic balance and the frequency-domain spectral balance analysis of nonlinear analog circuits. Simulations of a MESFET amplifier having one- and two-tone excitations were used to compare the performances of the GPSA–AOM and APDFT harmonic balance techniques. In general, based on the same accuracy consideration, the performance of the APDFT harmonic balance method is comparable to the GPSA–AOM with single-tone input excitation. The GPSA–AOM tends to dominate in circuits with two or more incommensurable signals. However, from the device-modeling viewpoint, most FDSB methods are limited to power-series-based models and thus have less utility than harmonic balance methods which use nonlinear models described by arbitrary functional relations.

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A New Resistance Measurement Technique Applicable to High-Temperature Superconducting Materials at Microwave Frequencies

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***Abstract* — A two-gap electrically floating resonant strip is used for surface resistance measurements of the high-temperature superconductor $YBa_2Cu_3O_{7-\delta}$. The method used is simple, has no electrical contact, operates at various resonant frequencies, and requires only a small sample. An analysis was used that allows for the accurate design of the strip dimensions to produce a desired resonant frequency. Experimental measurements on resonant frequencies in *X*- and *Ku*-bands (8–18 GHz) agree well with the calculations. The method allows one to extract the normalized**

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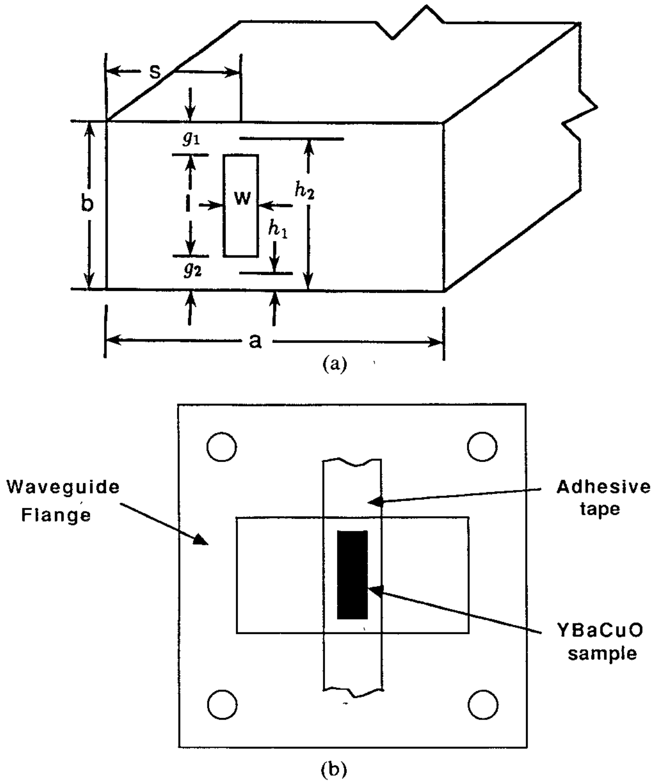

Fig. 1. (a) The two-gap strip structure used for analysis:

a	width of waveguide	w	strip width
b	height of waveguide	l	strip length
h_1	gap 1 position (center from bottom)	g_1	gap 1 size
		g_2	gap 2 size
h_2	gap 2 position (center from bottom)	$s' = s/a$	normalized strip position
		$w' = w/a$	normalized strip width
s	strip position (center from side)	$h' = h/b$	normalized gap position
		$g' = g/b$	normalized gap size
		$l' = l/b$	normalized strip length

(b) Superconductive sample supported by adhesive tape.

surface resistance of the sample from transmission coefficient measurements at the resonant frequency. These normalized values were found to compare favorably to the Mattis–Bardeen theory taken in the local limit. The resonant strip in waveguide should have applications in high-temperature superconductive material characterization and in the development of waveguide superconductive filters.

I. Introduction

$YBa_2Cu_3O_{7-\delta}$, with $\delta < 0.6$, is one of the newly discovered copper oxides that exhibit a superconducting transition at about 90 K [1]. The ease of achieving this temperature without the costly apparatus required for liquid helium temperatures, and the unique electrical and magnetic properties of a superconductor, make the YBaCuO compound an attractive candidate for microwave and millimeter-wave applications.

The most commonly used methods for the RF characterization of the high-temperature superconductors are cavity perturbation techniques [2], [3]. Others utilize disks [4] or stripline resonators [5]. The cavity techniques usually are useful only for single-frequency measurements, while the disk requires a rather large sample for characterization in the microwave range. The stripline method is complicated by the requirement of thin-film processing technology.

This paper reports a novel technique that may be used for measuring the resistive properties of high-temperature superconductors. The resonant structure consists of a superconductive strip that is supported in rectangular waveguide by a transparent (at microwave frequencies) adhesive tape of the type shown in Fig. 1. This method offers the advantages of simplicity, the elimination of electrical contact to the superconductor, and the possibility of operation at several frequencies. Additionally, the method requires relatively small sample sizes. Bulk polycrystalline epitaxial thin films or bulk single crystalline samples may be measured using the floating strip technique.

The losses due to the surface resistance of the strip are deduced by measuring the microwave transmission coefficient at the resonant frequency of the structure. The resonant frequency is shifted by varying the length of the superconductive sample.

An equivalent circuit representation of the floating YBaCuO strip in waveguide has been developed by incorporating the theory of Eisenhart [6] and a local case modification of the Mattis–Bardeen theory [7]. The calculated resonant frequencies and normalized surface resistance agree well with the measurements over a frequency range of 8 to 18 GHz. In particular, the frequency-squared dependence of the normalized surface resistance to frequency that is characteristic of the Mattis–Bardeen theory was observed.

In addition to its use for material characterization, this circuit also has potential for filter applications. The resonant strip forms the building block of waveguide filters. Low-loss waveguide filters can be developed by cascading the resonant strips in the longitudinal direction.

II. Circuit Modeling

The single-gap resonant post or capacitive strip structure has been analyzed by Eisenhart and Khan [8] and by Chang and Khan [9]. When applied to the characterization of high-T_c superconductors, this structure offers the benefit of small sample size and the potential for investigation at several frequencies within the bandwidth of the waveguide in use [10]. However, this method has revealed difficulties involved in making ohmic contact to the YBaCuO material. As a result, the structure was modified to a two-gap case where the resonant strip is isolated from the waveguide walls (Fig. 1).

The configuration shown in Fig. 1(a) has been studied by Eisenhart [6] as the mounting structure for two active devices. By combining Eisenhart's theory and the Mattis–Bardeen theory in the local limit, the equivalent circuit for a superconductive two-gap strip in a waveguide can be determined.

Eisenhart's analytical method for the reactance of the floating strip in a waveguide allows the design of a strip that produces a resonance at the appropriate frequency within the bandwidth of the waveguide. By doing successive experiments with resonances in *X*- and *Ku*-bands, it was hoped that the f^2 dependence of the surface resistance of the superconductive samples would be observed. Though Bardeen [11] has concluded that the characteristics of the high-T_c superconductors are probably not fully explained by the theory of Bardeen, Cooper, Schrieffer (BCS), Sridhar *et al.* [12] have shown that a slight modification of one of the parameters in the approximation gives a good match to existing surface resistance data for YBaCuO. Thus, the reactive component of the resonant structure used in this paper, the floating strip, consists of Eisenhart's analysis, while the resistive part consists of Sridhar's and Mattis–Bardeen's modification of BCS.

Fig. 2 shows the equivalent circuit of the floating strip. The calculation of the reactive elements (L and C) is straightforward and can be found by Eisenhart's analysis [6]. The calculation of the resistance is given in the following.

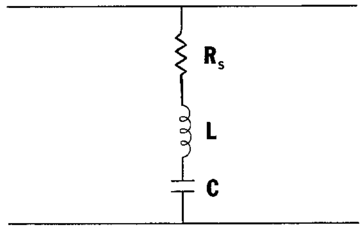

Fig. 2 Simplified equivalent circuit

Mattis and Bardeen [7] developed a theory for superconductors in ac fields using an extension of BCS. As in the two-fluid picture of superconductivity, conductivity is made up of normal superconducting components, in this case given by

$$\sigma = \sigma_1 - j\sigma_2 \tag{1}$$

where σ_1 is the normal and σ_2 is the superconducting component of the complex conductivity. The expressions for σ_1 and σ_2 in the extreme anomalous limit normalized by the normal-state conductivity σ_n of the material were derived by Mattis and Bardeen [7].

The normal-state surface resistance is

$$R_n = \sqrt{\frac{2\pi\omega}{\sigma_n c^2}} \tag{2}$$

where c is the speed of light and ω is the angular frequency. The normalized superconducting surface resistance becomes

$$\frac{R_s}{R_n} = \mathrm{Re}\left\{ \sqrt{\frac{2j}{\frac{\sigma_1}{\sigma_n} - j\left(\frac{\lambda_L}{\lambda_{\mathrm{eff}}}\right)^2 \frac{\sigma_2}{\sigma_n}}} \right\} \tag{3}$$

where λ_L is the London penetration depth and λ_{eff} is the effective penetration depth.

The $(\lambda_L/\lambda_{\mathrm{eff}})^2$ parameter would tend to be small for elemental materials such as tin or lead (< 5), and larger for compound superconductors in the extreme local limit (> 50) [12].

III. Experimental Results

A. Extraction of Impedance from S_{21} Measurements

Allowing for the elimination of connector and waveguide loss from the measurement calibration, the equivalent circuit of the strip obstacle can be represented by the simplified equivalent circuit shown in Fig. 2. The transmission coefficient, S_{21}, is related to the pure shunt admittance by [13]

$$S_{21} = \frac{2}{2+y} \tag{4}$$

where y is the normalized admittance.

The normalized strip impedance is

$$z = y^{-1} = \frac{R_s}{Z_0} + j\frac{X}{Z_0} = r_s + jx \tag{5}$$

where $r_s = R_s/Z_0$ and $x = (j\omega L - j(1/\omega C))/Z_0$. Hence, measurement of the magnitude and phase of S_{21} leads to the impedance presented by the YBaCuO strip. Unfortunately, the reactive component of this impedance so dominates the real part for all frequencies other than resonance that accurate values for R_s are difficult to obtain.

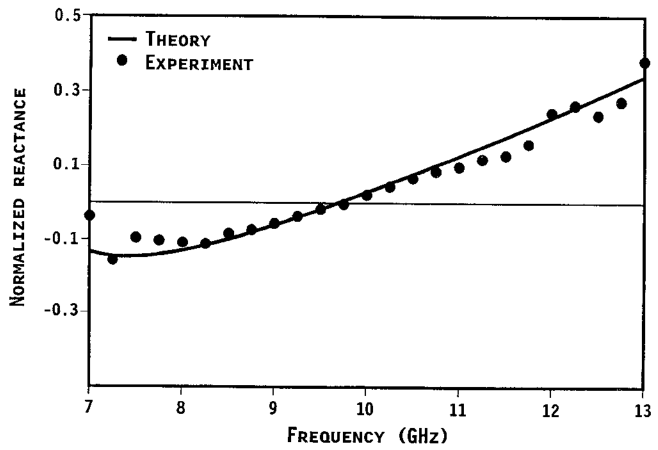

Fig. 3 Reactance of 10 mm YBaCuO strip in X-band waveguide with $a = 22.86$ mm, $b = 10.16$ mm, $w' = 0.08$, $l' = 0.98$, $s' = 0.50$, $g_1' = 0.008$ and $g_2' = 0.008$.

This problem was circumvented by calculating the real component of z from S_{21} data taken only at the resonant frequency. Since the reactive component of the impedance is zero at the resonance, the value of S_{21} is dictated solely by the resistive properties of the YBaCuO sample. At resonance, $z = r_s$ and from (4) we have

$$r_s = \frac{1}{2\left(\frac{1}{|S_{21}|} - 1\right)}. \tag{6}$$

Hence, the real component of the total impedance caused by the obstacle can be determined by the magnitude of S_{21} at the resonant frequency. This, however, requires that in order to characterize the resistance of the material over the entire bandwidth, the resonant frequency of the structure must be shifted. Since the resonant frequency of the floating strip in waveguide is dependent upon strip length, changing the dimensions of the YBaCuO sample will allow characterization of its resistance over most of X and Ku bandwidths.

B. Results

Essentially, two theories were combined in describing the impedance presented by an YBaCuO floating strip in waveguide: Eisenhart's two-gap analysis for the reactive components, and the modified BCS theory for the resistive. Theoretical results matched the measured results well, as is evidenced by Fig. 3, which shows the reactance of a 10 mm superconductive strip in X-band waveguide. Similar results were obtained in Ku-band for a 6.5 mm strip.

A comparison of the normalized surface resistance from the modified BCS theory with experimental data is shown in Fig. 4. R_s is measured at liquid nitrogen temperature and R_n at room temperature. Calibration of the network analyzer used for the S-parameter measurements was done at both temperatures. Three variables affect the positioning of the R_s/R_n curve: the operating temperature, the critical temperature of the YBaCuO, and the $(\lambda_L/\lambda_{\mathrm{eff}})^2$ parameter. The critical temperature was determined to be 87 K. The operating temperature was 78 K, based on measurements from a platinum resistive thermal detector. The remaining degree of freedom, $(\lambda_L/\lambda_{\mathrm{eff}})^2$, was then used to obtain the best fit to the data. As shown, the data trend matches that predicted by BCS for $(\lambda_L/\lambda_{\mathrm{eff}})^2 = 35$, considerably lower than the value of 65 used by Sridhar *et al.* [12]. The error bars on

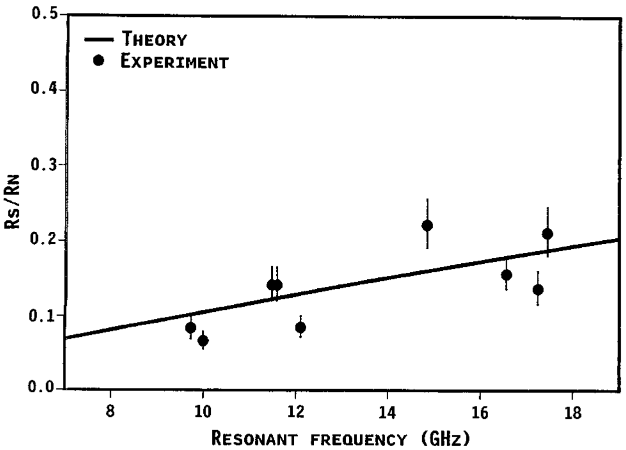

Fig. 4. Comparison of normalized surface resistance data with theory.

the data points were obtained by assuming an error of 1.7% in the S_{21} measurements. This error is due to the return loss of the coaxial to waveguide transitions, which is neglected in the impedance extraction method. A *VSWR* of 1.3 in the transitions will cause an error of 1.7% in S_{21} measurements.

IV. Conclusions

This paper reports a novel technique for measuring the resistive properties of high-temperature superconductors. It utilizes an analysis developed by Eisenhart for a two-gap, electrically isolated resonant strip in waveguide. Results of normalized surface resistance measurements show good agreement with a modified form of the Mattis–Bardeen extension of the Bardeen, Cooper, Schrieffer theory of superconductivity.

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Modal Analysis of Open Groove Waveguide

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Abstract —**A self-contained analysis for lower order modes in the open groove waveguide is presented. By adopting an approximate form for the fields in the outer region of the guide and then performing the remainder of the analysis rigorously, closed-form results for the fields and modal equation are obtained. The analysis allows a simple transverse network representation which can be compared with that obtained by Oliner and Lampariello. Universal dispersion curves, obtained numerically for the dominant mode, are presented.**

I. Introduction

The open groove guide has long been recognized as a low-loss waveguide which is most suitable for millimeter-wave circuits [1]–[5]. The low loss property is attributed to the absence of any dielectric material and to the low conductor losses, a condition which stems from the electric field lines of the dominant mode being parallel to the guide walls. Referring to Fig. 1(a), the electric field of the dominant mode is mainly parallel to the sidewalls. It is oscillatory in the inner region, $|y| < b/2$, and decaying in the outer regions, $|y| > b/2$. Because of the junction discontinuity at $y = \pm b/2$, a rigorous analysis of modes requires an expansion of the fields in terms of transverse modes in both the inner and outer regions. The process is lengthy [4] and the mathematical complexity will obscure any physical insight into the mode behavior. On the other hand, simple approximate solutions based on a single transverse mode field representation in each region (e.g. [1], [3]) will not have sufficient accuracy. More recently Oliner and Lampariello [6] have presented a simple and yet accurate solution based on an equivalent transverse network, taking into account the effect of higher order modes generated at the junction planes $y = \pm b/2$. Thus, the inner region is represented by a transmission line (Fig. 1(b)) connected through a step transformer to another line which represents the outer region. The effect of the higher order transverse modes is lumped into a susceptance B. By adopting results from [7], Oliner *et al.* [6] have deduced a useful formula for the susceptance B. They verify their results by comparing them with previous experimental data obtained by Nakahara and Kurauchi [1] and report favorable comparisons.

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